A topological approach to minimal change or
$n$-space cluster belief merging

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1 Introduction

In general, distance between two propositional logic valuations is calculated as the number of propositional symbols with different truth-values in the two valuations.

During information change operations, especially belief merging, one tries to find the set of valuations with certain properties (i.e., satisfying integrity constraints) which, in addition, have minimum aggregated distance with respect to models of the belief bases being merged. Whereas this process can be easily computed, it involves calculating the distances between every model of the integrity constraints and models of each one of the belief bases.

Our objective here is to see if there is a way of generating these models instead.

We start with representation of valuations in an $n$-dimensional space. Each propositional variable can be associated with one dimension of the space. For instance, for languages with one propositional variable the space has one single dimension with two points as seen below

\[ 0 \quad 1 \]

Two propositional variable languages have an associated bi-dimensional place with valuations possibly represented as follows:

\[
\begin{array}{c|c}
00 & 01 \\
\hline
10 & 11 \\
\end{array}
\]

Changes in the values of propositional variables are only allowed by following the edges. Each edge then corresponds to the flip of one truth-value of a propositional variable between two valuations. Three-propositional variables have three-dimensions as shown below.
In general, for $n$ propositional variables, $n$ dimensions will be necessary. Mathematically, this is easy to devise, although it is somewhat difficult to visualise.

In four dimensions, we get what is called a tesseract. The figure below represents an orthogonal projection a tesseract where the nodes are valuations of a 4-variable propositional language.

An alternative way of thinking of the distance is to consider meta-levels: the flips of values take you to dimensions of ever lower level. For instance, four-dimensional spaces would be represented as
Whichever way you look at it, each valuation is actually a simply point in the \( n \)-dimensional space, which brings us back to our problem:

- what are the points in this \( n \)-dimensional space that are closest to a collection \( \mathcal{C} \) of sets of other points?

The traditional way of looking at this problem, as mentioned before, is to look at the set of all points in the space and calculate its distance to the collection individually, but the \( n \)-dimensional approach suggests that there may be a subspace whose points are the only relevant ones in the calculation.

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**NOTE TO AUTHORS:**

Is it possible to represent all valuations in a three-dimensional space? This can be also be formulated in the following way:

- Given an infinite but countable set of objects with a distance function \( d \) which is a metric, is it possible to map these objects into a 3-dimensional vector such that for any two objects \( o_1 \) and \( o_2 \) there are points \( p_{o_1} \) and \( p_{o_2} \) such that \( d(o_1, o_2) \) is equal to the Euclidean distance between \( p_{o_1} \) and \( p_{o_2} \)?